Grade 9/10 Math Circles

February 22

The Shape of You - Problem Set

Graph Colouring

1. Try to color the map of the United States so that no two neighbouring states have the same color, **using at most four colors**. Note that coloring maps is the same as coloring graphs (why?)

2. Prove the **Handshake Lemma** from the lecture notes.

Series Examples

3. Prove the **Geometric series** formula from class:

$$1 + r + r^2 + r^3 + \ldots = \frac{1}{1-r}$$

4. What should the sum

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots = ?$$

equal? Is it finite or infinite?

5. What should the sum

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots = ?$$

equal? Is it finite or infinite?

Three-Dimensional Fractals

6. Try drawing (or building) Sierpinski's Gasket in three dimensions!

7. Try drawing (or building) Sierpinski's carpet in three dimensions!